Scaling of hysteresis in the Ising model and cell-dynamical systems in a linearly varying external field

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We present results from simulations of the hysteresis loops in the two-dimensional (2D) Ising model and a cell-dynamical system (CDS) in a linearly, rather than sinusoidally, varying external field. We find in the CDS a disorder-induced transition, which has behavior similar to the critical point in the 2D Ising model. Below the critical point, the area of the hysteresis loops, representing the dissipation per cycle, scales with the rate of the driving field $\dot{H}$ as $A = A_0 + a\dot{H}^{\alpha}$, with a nearly constant $\alpha \sim 0.36 \pm 0.08$ for the Ising model and 0.66 ± 0.02 for the CDS. Thus, the CDS belongs to the class of mean-field models, which is different from that of the Ising model. Above the critical point, both the Ising model and the CDS give $A_0 = 0$ and an α that increases with temperature and disorder.

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I. INTRODUCTION

Recently there is increasing interest in phase transitions driven by a time-dependent external field. Theoretical [1–10] and experimental [11,12] results have demonstrated that the energy dissipation, represented by the area of the hysteresis loops A, can be cast into a scaling form

$$A \approx H_0^{\alpha} \Omega^{\beta} \tag{1}$$

for low amplitude H_0 and frequency Ω of the sinusoidally varying field. The first theoretical results [1] on the large-N model gave $\alpha \sim 0.66$ and $\beta \sim 0.33$. It is now clear, however, from both numerical [3,6,9] and analytical [3,6] outcomes, that the exponents should be $\alpha = \beta \sim 0.5$, since in the large-N model the two phases with opposite magnetization are connected by continuous paths that can circumvent the barrier between them owing to the continuous symmetry. Numerical simulations on discrete models with scalar order parameters are also at hand. The Monte Carlo simulation of the two-dimensional (2D) Ising model by Lo and Pelcovits (LP) [2] gave rise to the exponents $\alpha \sim 0.46 \pm 0.05$ and $\beta \sim 0.36 \pm 0.06$ in narrow ranges of H_0 and Ω. The simulation of a cell-dynamical system (CDS) for the time-dependent Ginzburg-Landau equation for a scalar 2D $(\Phi^2)^2$ model by Sengupta, Marathe, and Puri (SMP) [4] yielded $\alpha \sim 0.47 \pm 0.02$ and $\beta \sim 0.40 \pm 0.01$ in wider ranges of the parameters. It was claimed to be in accordance with the Ising model and to belong to the same universality class. However, it is fitted to a four-parameter form $ax + bx^2$ with a larger $b = 30.7$ than $a = 5.9$, where x is given by Eq. (1). In fact, the double logarithm plot of the area of the hysteresis loops versus H_0 for different frequencies in Fig. 3 of SMP clearly shows that there is no single exponent α, since it depends on Ω.

Motivated by the experimental techniques of linear driving fields both in internal friction [13] and in differential scanning calorimeter (DSC) [14] measurements, which also lead to power law variation of energy dissipation with the scanning rate of the driving field, we have investigated systematically the energy dissipation both in magnetic [9] and in thermal [10] hysteresis with the rate of the linear driving field in the large-N model. Results show that the relation between the energy dissipation per cycle and the rate of the field, $\dot{H}$, is divided into two classes. One has

$$A \sim \dot{H}^{1/2} \tag{2}$$

which includes the hysteresis loops with continuous paths circumventing the barrier between the two phases, like the model with O(N) symmetry. The other, the mean-field class, appears to be

$$A = A_0 + a\dot{H}^{2/3} \tag{3}$$

which covers primarily the mean-field models [11,9], as well as the thermal hysteresis loops [10] and the double hysteresis loops [9] in the large-N $(\Phi^2)^3$ model, where A_0 and a are constants. In this class, the transition can only take place beyond the spinodal point, where the energy barrier vanishes. As a consequence, there is a finite dissipation even at zero scanning rate. Thus it is desirable to investigate discrete models with a scalar order parameter both to resolve the discrepancy and to search for new classes.

In this paper, we perform simulations of the 2D Ising model (Sec. II) and CDS (Sec. III) of LP and SMP, with a linear, rather than a sinusoidal driving field. Results show that they belong to different classes in contrast to the previous results. The Ising model belongs to a new class, whereas the CDS belongs to the mean-field class. We also find in the CDS a disorder-induced transition, which has similar behavior to the critical point in the 2D Ising model. In other words, above the transition point, the scaling exponent for both models increases significantly with the temperature and disorder, while below it the exponent is nearly constant.

II. ISING MODEL

Consider a 2D Ising model on a square lattice with a linear sweeping external magnetic field H'. The Hamiltonian is given by

$$\mathcal{H} = -J' \sum_{\langle ij \rangle} s_i s_j - H'(t) \sum_i s_i \ , \tag{4}$$

where $s_i = \pm 1$, J' denotes the coupling constant, and $\langle i,j \rangle$ the nearest-neighbor pairs. We employ the standard Metropolis Monte Carlo algorithm [15] and periodic boundary conditions. Dimensionless parameters will be used below for clarity, and hence the temperature is given by the inverse coupling $J = \beta J'$ and the magnetic field $H = \beta H'$, where $\beta = 1/k_B T$. Thus the critical temperature is reached at $J = J_c = -\ln(1-\sqrt{2})/2 \approx 0.44$ [16].

Given a coupling j and a field sweeping rate $\dot{H}$, we chose a sufficiently large initial field to saturate the system in one direction, and then cyclically sweep the field in a sawtooth way to obtain hysteresis loops. The time unit is one Monte Carlo step, within which all the spins are flipped sequentially in an identical external field following the Monte Carlo algorithm. The lattice size we used is 50×50, which is sufficient for the statistics of the data obtained, though insufficient in the case of sinusoidal driving [1]. Up to 100×100 lattice size is also used, and the results are found to be independent of the lattice size. This has also been found previously [2,7].

Generic hysteresis loops are shown in Figs. 1(a) and 1(b), for the temperature below and above the critical point, respectively. The area of the hysteresis loops versus the field sweeping rate is plotted in Fig. 2 at several temperatures. The hysteresis loops below and above the critical point are quite different. Above the critical point, only sufficiently large sweep rates can give rise to hysteresis loops, whereas below it there is a finite area even for very small rates. This manifests itself clearly in the scaling of the area with the field sweep rate, i.e., the area can be best fitted to

$$A \sim A_0 + a\dot{H}^{\alpha} \ , \tag{5}$$

with a finite A_0 for $J > J_c$ or $T < T_c$, while $A_0 = 0$ for $J < J_c$. Moreover, for $J > J_c$, α approximates to 0.36 ± 0.08, only weakly dependent on J, whereas it increases significantly with decreasing J below J_c, so that in the latter case no single exponent α exists. This result is quite different from that of Ackaryya and Chakrabarti [7], who found no signature across the critical point. The exponent 0.36 ± 0.08 is also compatible with the result of LP within the statistical errors.

It is interesting that the simulation results reveal a nonzero A_0 for the zero rate below T_c. In principle, fluctuations of any amplitude have an infinite amount of time to induce a transition over the free-energy barrier as the ramp rate approaches zero. One cannot wait an infinitely long time to check this result in the simulation, as well as in real experiments that are plagued by hysteresis. However, the loop areas of very small rates do deviate systematically from converging to zero, extrapolating to a finite A_0. Thus it is still not conclusive whether the nonzero loop area A_0 results from the finite observed time or from other intrinsic mechanisms. Our results, including the CDS ones below, as well as many real transitions, the nonisothermal martensitic transformation, for instance, seem to support the latter.

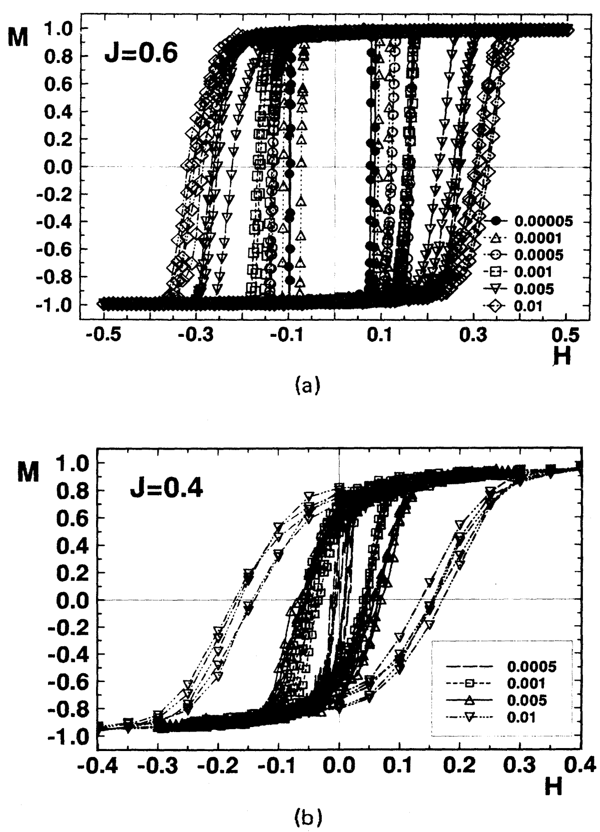

FIG. 1. Hysteresis loops of the Ising model at temperatures below (a) and above (b) the critical temperature. The numbers indicate the sweeping rate of the external field H.

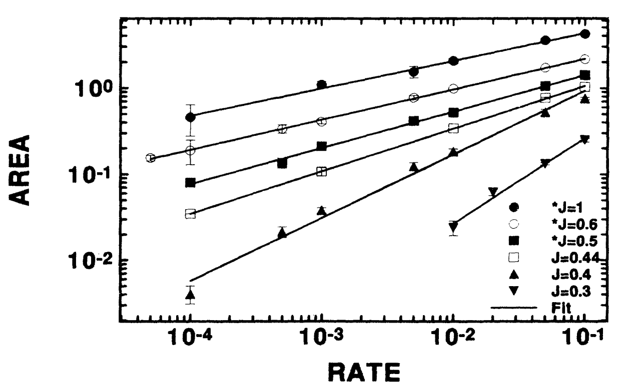

FIG. 2. Area of the hysteresis loops vs sweeping rate of the Ising model at various temperatures indicated in the legend. The straight lines are best fits to the data. Data points without error bars have negligible errors. Note that A_0 has been deducted from the area of those data with temperature below the critical temperature (indicated by a star before J).

It is reasonable that different behavior has been found in the hysteretic response below and above the critical point, as well as in the Ising model and the large-N model when $T < T_c$, since below T_c the scalar Ising model should overcome the transition barrier between the two phases, while the large-N model has symmetry paths that can simply circumvent the barrier. The difference between the Ising model and the mean-field model may probably arise from the fluctuations and short range interactions in the former, so that it is easier for the system to flip from one phase to the other, leading to a smaller α.

When $T > T_c$, on the other hand, the fluctuations of individual spins are so strong that little correlation persists between different spins. Accordingly each spin flips independently. In contrast to the fluctuations that can trigger various scales of avalanches below T_c, these independent fluctuations above T_c dissipate extra energy. As a result, the higher the temperature, the stronger the fluctuations and hence the bigger the α.

III. CELL-DYNAMICAL SYSTEMS

A cell-dynamical system [17] is a space-time discrete dynamical system with a continuous variable defined on each lattice point. This variable evolves according to some rules that relate it to the states at previous time steps. The CDS has been devised as a computationally efficient method to model phase separation kinetics. It is essentially a space-time discretized version of the full time-dependent Ginzburg-Landau equation.

We use a 2D square lattice with the following update rule for the variable Φ_i at site i in the case of nonconserved order parameter [17,4]:

$$\Phi_i(t+1) = P\tanh[\Phi_i(t)] + D\mathcal{L}[\Phi_i(t)] + B\eta_i(t) + H(t) , \tag{6a}$$

where P $(=1.3)$ and D $(=0.5)$ are constants, chosen to lie within the stability regime to avoid artifacts [17], $\eta_i(t)$ is a Gaussian white noise with zero mean and unit variance, and B is a constant representing the amplitude of fluctuations or disorder and can be considered as proportional to temperature (not necessarily linearly). $\mathcal{L}[\Phi_i(t)]$ is essentially the isotropized discrete Laplacian with the following definition for the 2D square lattice:

$$\mathcal{L}[\Phi_i(t)] = \tfrac{1}{6}(\text{summation of } \Phi \text{ in the nearest neighbors}) + \tfrac{1}{12}(\text{summation of } \Phi \text{ in the next-nearest neighbors}) - \Phi_i(t) . \tag{6b}$$

It has been checked that the presence of the next-nearest neighbors in Eq. (6b) is irrelevant to our results. Periodic boundary conditions are also applied, and the lattice size we used is also mainly 50×50. The results have been checked as in the Ising model to be independent of the lattice size, as observed by SMP too.

Hysteresis loops are obtained as in the Ising model, the only difference being that, as for cellular automata [18], the update procedures for the CDS are parallel or synchronous. The magnetization $M(t)$ is acquired as the average of Φ over the whole lattice for every corresponding value of $H(t)$. Generic hysteresis loops are shown in Figs. 3(a) and 3(b) with different disorder B. The area of the hysteresis loops versus the scanning rate of the external field is presented in Fig. 4.

Comparing with the hysteresis loops of the Ising model, it can be seen that there is a transition similar to the critical point of the Ising model, arising from the different amplitude of the disorder B. The behavior of the hysteresis loops below and above the critical point $B_c \sim 0.45\pm 0.01$, as well as the scaling of the area, are all similar to the Ising model. The only difference is that $\alpha \sim 0.66\pm 0.02$ for the CDS, and thus it belongs to the mean-field class according to Eq. (3). This is possibly because a mean-field-like average is involved in the CDS, so that fluctuations are effectively reduced.

There is one more difference between the Ising model and the CDS when the temperature is sufficiently low and thus fluctuations are effectively reduced. For the Ising model, J is thus extremely large and the transition can only take place near $4J$, almost independent of the sweep-

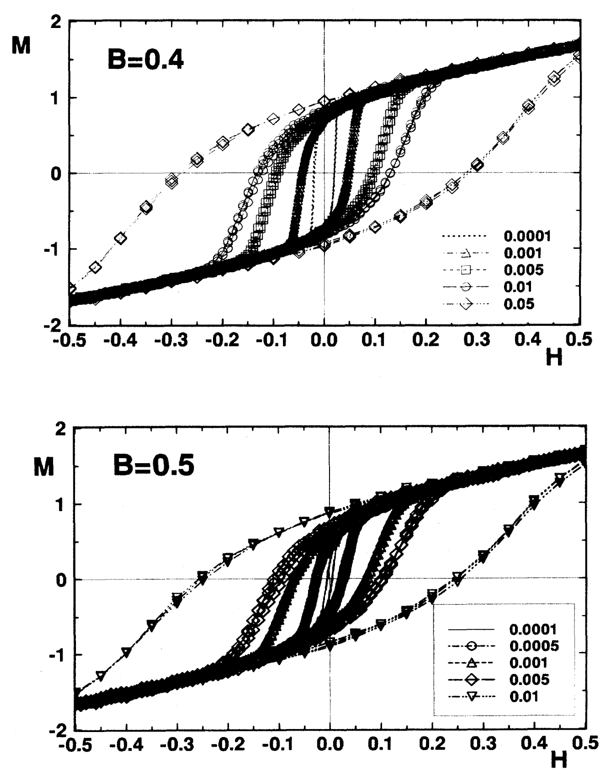

FIG. 3. Hysteresis loops of the cell-dynamical system at disorder B below (a) and above (b) the critical point. The numbers indicate the sweeping rate of the external field H.

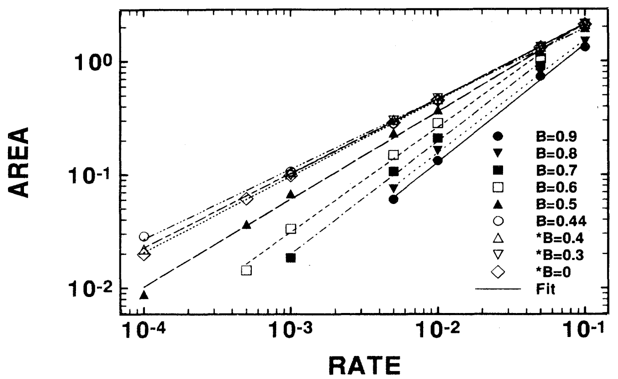

FIG. 4. Area of the hysteresis loops vs sweeping rate of the cell-dynamical model at various amplitudes of the fluctuation B indicated in the legend. Errors are smaller than the sizes of the symbols and are not shown. The lines and stars have the same meaning as in Fig. 2.

ing rate; consequently $\alpha\sim 0$. Therefore the results of Sec. II are only for not sufficiently low temperature, otherwise α would decrease to zero. In contrast, for the CDS, no peculiarity has been found for $B=0$ as can be seen in Fig. 4.

The transition induced by the fluctuation B is akin to a dynamic phase transition [19,1,2,4]. Nevertheless, it is associated with $\dot{H}\rightarrow 0$, while the dynamic transition is obtained by changing the amplitude of the oscillating field at fixed frequency and disorder B. It is much more similar to the usual equilibrium order-disorder transition induced by temperature, which increases the competence of entropy. However, almost no finite-size dependence has been found. A detailed study of the transition is out of the scope of the present paper.

IV. SUMMARY

We present results from the simulations of hysteresis loops in the two-dimensional Ising model and a cell-dynamical system under a linear rather than a sinusoidal driving external field. We find in the CDS a transition induced by the amplitude of the fluctuation near 0.45 ± 0.01. The transition has similar behavior to the critical point in the 2D Ising model. Since scalar order parameters are encountered in both models concerned, a transition free-energy barrier between the two phases should be surmounted even for vanishingly small ramp rates. In contrast, a system with vector order parameters can simply circumvent the barrier via the continuous paths. As a result, below the critical point, a nonzero dissipation ensues for vanishing sweeping rates, and the area of the hysteresis loops, representing the dissipation per cycle, scales with the rate of the driving field $\dot{H}$ as Eq. (5) with a nearly constant $\alpha\sim 0.36\pm0.08$ for the Ising model in a certain range of the temperature and 0.66 ± 0.02 for the CDS. Thus the CDS and the Ising model belong to different classes, in contrast to what has been found previously. The CDS belongs to the mean-field class, while the Ising model constitutes a new class. Above the critical point, on the other hand, both models give $A_0=0$ and an α that increases with the temperature and disorder.

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